Advances in Dynamic Risk Budgeting: Efficient Control of Absolute and Relative Risks †

Daniel Mantilla-Garcia⋆ and Hugo Lestiboudois *

⋆*Koris International, Head of Research & Development*
**Koris International, Research Assistant*

July 2012

†Koris International is Investment Advisor, registered under number D011872 by the CNCIF, association approved by the French Regulator (AMF). Contact details are: **email**: daniel.mantilla@koris-intl.com, **Address**: Koris International, Parc Technologique de Sophia-Antipolis, 200 avenue de Roumanille, Immeuble "Neri", 06410 Biot - France. **Phone**: + 33 (0)4 88 72 88 30.

Advances in Dynamic Risk Budgeting: Efficient Control of Absolute and Relative Risks

Abstract

In the aftermath of the 2008 crisis, investment strategies using maximum drawdown as a risk management objective have gained popularity. A systematic approach to control the maximum drawdown is the TIPP strategy, which allows investors to limit drawdowns to a chosen pre-defined level or risk-budget by dynamically allocating wealth between the "risk-free" asset and a performance-seeking portfolio. However, a weak interest rate environment represents a burden on this kind strategy, due to an increased opportunity cost for the investor. This article illustrates innovative variants of the maximum drawdown control technique, seeking to decrease the opportunity costs of the drawdown protection as well as variations of the strategy to control maximum underperformance of benchmarked portfolios, opening new perspectives for investors in terms of loss-controlled investments.

In the post-2008 era, a robust risk management process has been recognised as the basis to successful investment among best practices. In a highly uncertain environment, risk management processes should somehow address one of the main concerns of investors, namely, value protection.

Loss aversion is a well documented feature of investors. Thus limiting maximum drawdown as a risk management objective is gaining popularity. This risk-metric turns out to be more intuitive to investors than volatility, for whom risk indicators should include potential cumulative losses on a portfolio.

However, the current weak interest rate environment has weighted on "absolute return" investment approaches, inciting institutional investors to shift their focus towards more efficient risk management practices that do not enter in conflict with performance objectives.

In this article, we propose dynamic risk-budgeting techniques to address the need for more cost-efficient (absolute) loss-controlled strategies. Furthermore, we introduce the concept of "relative" maximum drawdown[1] (or maximum cumulative underperformance) and a dynamic allocation strategy that limits cumulative underperformance with respect to a given (investable) benchmark.

This type of investment strategy can be of particular interest for institutional investors to enter alternative attractive investments such as emerging markets or high yield bonds, that come with an important "benchmark-risk" component. For example, while emerging market indices have outperformed developed large-cap equity indices on the long run and are expected to keep heading in the same direction, they have severely underperformed developed equity indices on several occasions. This phenomenon is particularly noticeable during market rallies, where relative cumulative underperformance can reach more than 50%[2] over certain periods.

[1] Not to be confused with the standard maximum drawdown (MDD) concept, which is sometimes called relative MDD to differentiate it from its version expressed in dollar terms as opposed to relative change (returns) terms. In this article we always work with drawdowns expressed in percentage terms.

[2] See Table 4 and Figure 1 for details.

Avoiding the Cash Trap of the Maximum Drawdown Control

A systematic approach to controlling the maximum drawdown was introduced by Estep and Kritzman (1988)[3] with the Time-Invariant Portfolio Protection (TIPP) method, which prevents the portfolio from experiencing losses in excess of a pre-set limit at all times. The MDD is defined as the largest drop in value from a peak to a bottom, observed at an instant t. More precisely, for a value process $V(t)$, the maximum drawdown from time t_0 to current time t is given by:

$$\overline{D}_{t_0,t}(V) = \max_{s\in(t_0,t)} -D_s(V)$$

$$\text{where } D_t(V) = \frac{\max_{s\in(t_0,t)} V(s)}{V(t)} - 1$$

where $V(t)$ represents the value of the portfolio at time t and the drawdown $D_t(V)$ is the percentage loss experienced by asset V at time t with respect to its running maximum. In order to limit the MDD in a portfolio, one would set a Floor value as a function of the risk budget x, which represents the maximum percentage of the current capital the investor is willing to lose on the portfolio at any time. The floor would then be defined at time t as:

$$F(t) = k \times \max_{s\in(t_0,t)} V(s)$$

where $k = (1-x)$. In order to limit the MDD of the portfolio to x at all times, the TIPP strategy has to be implemented within portfolios composed of a performance-seeking component and a reserve asset. The allocation between the two components depends on a multiplier m, and a "cushion". The allocation to the performance-seeking (PSP) component at time t being:

$$W_{PSP}(t) = m_t \times [V(t) - F(t)].$$

In the event of a slump in the performance-seeking component level, wealth is reallocated towards the reserve component to prevent the portfolio from breaching its Floor value. As the Floor is a strictly increasing function of time, the reserve asset has to be an asset with a strictly positive performance at all times, thus bounded to be cash. Despite enabling the portfolio to fulfil its risk-management objective, the strategy may find itself having an important proportion invested in cash during a long and indefinite period of time, which increases the opportunity cost of the strategy. We thus proceed to a generalization of the MDD floor.

The generalization of the MDD measure and the corresponding floor setting has to do with the introduction of a look-back period. We restrict the measurement of the MDD measure to a given look-back or "trailing" period of time instead of calculating it since inception[4]. This changes the risk-control objective: instead of controlling the MDD over the life of the investment, the strategy limits the maximum loss over a given rolling period. This presents three major advantages for closed-end funds and open-end funds with a recommended investment horizon:

[3] Tony Estep and Mark Kritzman. "TIPP". in: *The Journal of Portfolio Management* 14.4 (1988), pp. 38–42

[4] From there on, we will refer to the traditional maximum drawdown as the "global MDD", and to the maximum drawdown with a restricted calculation horizon as the "trailing MDD".

- The first advantage of using a trailing MDD Floor is that it enables a reload of the risk budget to avoid being locked in the reserve asset (cash) for an indefinite amount of time.

- The second interesting aspect of a trailing MDD Floor is that it enables a guarantee on the portfolio value net of management fees. Management fees over the Floor horizon can be anticipated in the initial risk budget. Without such a horizon, the MDD limit cannot be guaranteed if the risk-free rate is lower than the fees rate.

- The strategy with this Floor locks-in $k\%$ ($k < 100\%$) of the realised gain over the trailing period. In other words, a guarantee on $k\%$ of the performance cumulated over the trailing period can be offered to the investor.

In Table 1, we present the summary statistics of implementing a portfolio controlling for 5-year trailing MDD to a portfolio controlling for global MDD, both of them being exposed to US Large Cap stocks[5] during the period 06-1926 to 05-2012. During the overall historical period, the portfolio with the global MDD control has 164 months with an allocation to the stock index of less than 10%, compared to only 23 months for the portfolio with the 5-year-trailing MDD floor. This is a very significant reduction of particular interest during low risk-free rate periods. Consistent with this result is the difference in average allocation to the stock index, of 41% for the global MDD control strategy compared to 45% for the trailing MDD one.

The strategy with a trailing MDD Floor uses the risk budget more efficiently as depicted by a higher Calmar[6] ratio of 0.48 compared to 0.42 for the global MDD strategy, resulting from higher excess returns over the 1-month US Treasury Bills benchmark for the trailing MDD portfolio of 3.65% versus 2.76%. It is to be noticed that, while the global MDD of the trailing MDD strategy is slightly above the 15% risk budget, the trailing MDD over the 5-year trailing period is 14.73%.

Table 1: *Performance comparison of a long-only US Large Cap portfolio and dynamic risk-budgeting strategies controlling respectively for global MDD and 5-year trailing MDD for the period 06-1926 to 05-2012. The reserve asset is proxied by the one-month US Treasury Bills rate. Return and volatility statistics are annualized.*

	US Large Cap	Global MDD	5Y Trailing MDD
Return	9.56%	6.32%	7.21%
Volatility	18.85%	6.56%	6.86%
Excess Return	6.00%	2.76%	3.65%
Trailing MDD	65.84%	13.69%	14.73%
Global MDD	83.72%	15.08%	15.17%
Calmar Ratio	0.11	0.42	0.48
Months with $W_{PSP} < 10\%$	-	164	23
Average W_{PSP}	-	41%	44%

[5]Refer to data sources section in "End Notes" section for details on large cap portfolio composition.
[6]Calmar ratio is defined as the ratio of return p.a. over maximum drawdown.

Reducing Protection Cost with the Excess Maximum Drawdown Floor

A wide number of loss-averse investors retain the MDD as the most appropriate measure of risk. However, the standard MDD control strategy, i.e. the TIPP, embeds the restriction of using cash as the reserve asset. In the presence of low interest rates and a relatively low risk budget, this restriction can produce strategies with unattractive expected returns.

Furthermore, institutional investors such as pension funds facing funding ratio constraints cannot afford to invest an important part of their portfolio in cash, as they need to hedge their long term commitments with long duration fixed-income instruments. In other words, their reserve asset must be different from cash.

To address this issue, we introduce a new type of floor that limits what we call the excess maximum drawdown (EMDD). The excess drawdown of a portfolio at time is simply defined as the difference between the portfolio's maximum drawdown and its reserve component's maximum drawdown. More precisely, for a portfolio A and a reserve asset B, excess maximum drawdown from time t_0 to current time t is given by:

$$EMDD_{t_0,t}(A,B) := \overline{D}_{t_0,t}(A) - \overline{D}_{t_0,t}(B)$$

$\overline{D}_{t_0,t}(A)$ and $\overline{D}_{t_0,t}(B)$ representing the portfolios' maximum drawdown at time t. The floor is then designed so as to maintain the EMDD of the portfolio under the risk budget of $x\%$.

This new floor allows investors to limit cumulative losses incurred by the performance-seeking asset while still enabling the use of assets other than cash in the reserve compartment. Using this loss-controlled strategy, the reserve asset can be composed for instance of a portfolio with a known MDD (e.g. using as reserve asset a portfolio with MDD control), or of assets with a "tolerable" level of expected MDD, such as a bond portfolio (for pension funds the reserve asset might be the Liability-Hedging Portfolio).

Table 2: *Performance comparison of the MSCI EM and one-year trailing EMDD strategy for the period 31/12/1998 to 31/01/2012. The reserve asset is the J.P. Morgan EU Emerging Markets Bond Index.*

	JPM EU EMBI	MSCI EM	1Y Trailing EMDD
Return	8.11%	13.07%	17.35%
Volatility	6.23%	20.12%	14.54%
Trailing MDD	12.16%	49.00%	24.32%
Excess Return to Benchmark	-	4.97%	9.24%
Excess Volatility to Benchmark	-	13.89%	8.31%
Trailing EMDD	-	36.84%	12.16%
Excess Calmar Ratio	-	0.35	1.43

This option is not available with the standard TIPP strategy because the MDD floor cannot be hedged with reserve assets other than cash.

Table 2 presents the results of backtesting a one-year-trailing EMDD floor portfolio during the period 31/12/1998 to 31/01/2012, using emerging markets bonds as the reserve asset, and emerging markets equities as performance-seeking asset and set the risk budget to 15%. The strategy displays a maximum drawdown in excess of the JPM EU Emerging Markets Bond Index MDD of 12.16%, which is under the 15% risk budget set ex ante. Despite this risk-management constraint, this strategy yields an impressive annualized absolute return of 17.35%, well above the 13.07% of the performance-seeking asset, i.e. the MSCI Emerging Markets equity index. The strategy controlling for excess maximum drawdown widely outperforms the MSCI EM equity index also in terms of risk-adjusted returns, displaying significantly lower MDD and volatility figures. As a consequence, the trailing EMDD floor portfolio's "excess Calmar ratio", is over four times higher than that of the MSCI EM index.

A portfolio with a trailing EMDD floor enables a controlled exposure to emerging markets, providing interesting absolute and risk-adjusted performances when compared to its corresponding bond and equity components over the period. The value-loss risks of going long on an all-equity emerging market portfolio are kept under control, while the equity premiums are captured in a return-efficient manner by using emerging market bonds to limit equity-induced losses as opposed to cash.

In Table 3, we summarize the characteristics of the Excess Drawdown metric, which we have just introduced. It provides the reader with a general view of how it is measured, what it evaluates and when it attaints its maximum value. The exhibit also compares it to the Relative Drawdown statistic, which we introduce in the next section.

Seeking Excess Returns with Benchmark Underperformance Control

In turbulent markets, severe underperformance relative to a given benchmark is one of the main risks faced by portfolio managers and institutional investors seeking upside potential. While pension funds must control the underperformance with respect to their liabilities, asset managers might control the underperformance of active investment strategies with respect to passive ones, such as cap-weighted equity indices.

To control for benchmark risk, we measure what we call Relative Maximum Drawdown (RMDD), defined as the maximum cumulative underperformance of a portfolio relative to a given benchmark.In other words, it measures the maximum relative loss of the portfolio with respect to the benchmark.

Table 3: *Comparison of the characteristics of the Excess Drawdown and Relative Drawdown statistics.*

Excess Drawdown	**Relative Drawdown**
Difference between portfolio's total drawdown and reserve/reference asset's drawdown.	Maximum cumulative underperformance (relative loss) w.r.t. benchmark.
Measures absolute losses	Measures relative losses.
Maximum value occurs during a drawdown of the portfolio (thus most likely in a "bear" market).	The maximum value may be reached in a bull market (if the benchmark presents its largest outperfomance relative to the portfolio during that period).
The EMDD is associated to the same period of time as the MDD of the portfolio.	The RMDD is independent of the (absolute) MDD, thus it may or may not occur over the same period of time.

Considering a portfolio A and its benchmark B, we measure the RMDD of the portfolio relative to its benchmark at time t in the following way:

$$\overline{RD}_{t_0,t}(A,B) = \min_{t^\star \in (t_0,t)} \{R_A(t^\star,t) - R_B(t^\star,t)\}$$

where $R_A(t^*,t)$ and $R_B(t^*,t)$ stand for the returns of the portfolio and its benchmark between time $t^\star$ and t for any $t^\star < t$. Using the (investable) benchmark as the reserve asset and a dynamic allocation to the outperformance-seeking asset, according to the corresponding risk budget, the portfolio's relative drawdown can be constantly kept within a predefined risk budget. What The maximum RMDD strategy does, as opposed to traditional CPPI strategy, is that it locks-in realised alpha while keeping the eventual benchmark underperformance inferior to the risk budget *at all times.* The RMDD floor can limit the typical temporary underperformance relative to the benchmark that even skilled active managers are confronted with during strong bull markets, significant rebounds or turning points.

This floor can be used to facilitate the access to alternative risk premiums such as emerging markets, even in hard-to-forecast markets, without underperforming the reference developed market benchmarks by more than a pre-set maximum tolerated level at any point in time.

Table 4 reports the results of Backtesting a one-year-trailing RMDD strategy using a 15% risk budget from 1987 to 2012, investing in the MSCI World (the benchmark in this experiment) and the MSCI EM index as the outperformance asset.

Table 4: *Performance comparison of the MSCI EM and a dynamic risk-budgeting strategy controlling for one-year trailing RMDD for the period 31/12/1987 to 31/01/2012. The reserve asset and benchmark is the MSCI World equity index. All return and volatility statistics are annualised.*

	MSCI World	MSCI EM	1Y Trailing RMDD
Return	7.57%	13.37%	13.11%
Volatility	15.47%	24.23%	19.04%
Global MDD	51.91%	59.18%	56.98%
Excess Return to Benchmark	-	5.81%	5.54%
Excess Volatility to Benchmark	-	8.76%	3.56%
Relative Maximum Drawdown	-	51.68%	14.32%
RMDD Period	-	Sep 1994-Aug 1998	Dec 1993-Jan 1998
Relative Calmar Ratio	-	0.259	0.915

Figure 1: *Representation of the Relative Drawdown of the MSCI Emerging Markets index and the Relative Risk-Control strategy with respect to their benchmark, the MSCI World index, from 31/12/1987 to 31/01/2012. The Relative Risk-Control strategy limits the RMDD relative to its benchmark to a maximum level of 15% in this experiment.*

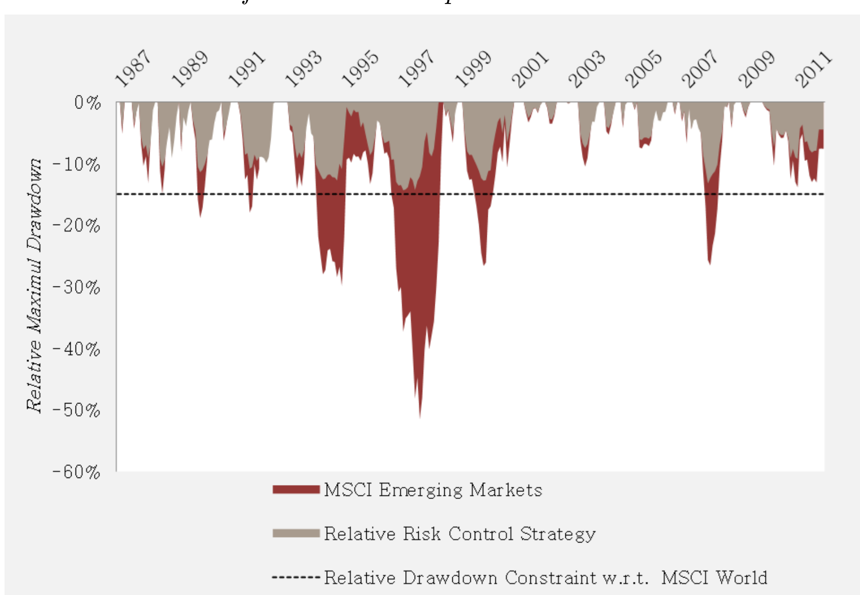
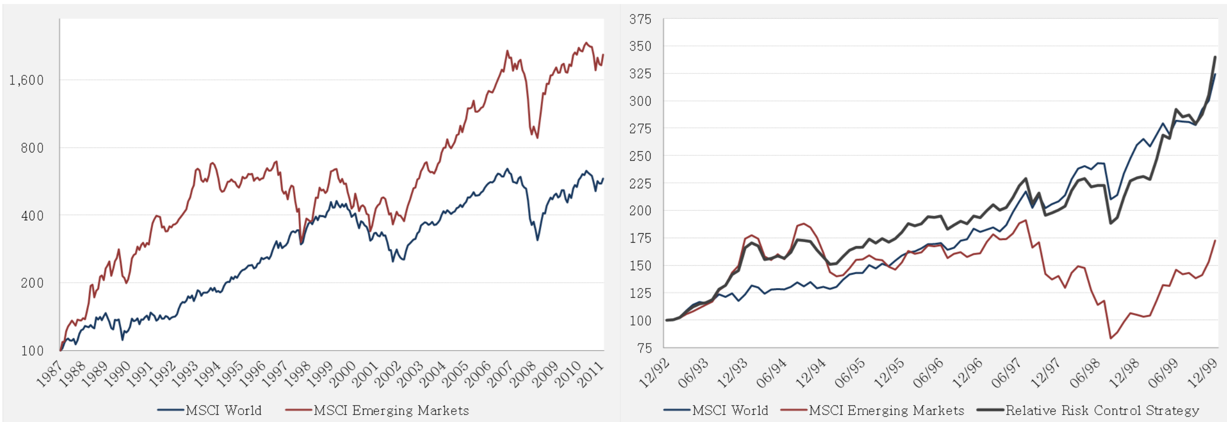


Figure 2: *This figure illustrates a phase during which an emerging markets drawdown episode coincides with a developed markets rally. This demonstrates the benefits of a Relative Drawdown Control technique when seeking to get exposure to alternative markets*



While the RMDD (maximum cumulative underperformance) of the MSCI EM relative to the MSCI World reaches an impressive -51.68%, the benefits of the risk-control really stand out, as the strategy respects its 15% risk budget, presenting a RMDD of 14.32% over the period, while still providing an annualised outperformance over the MSCI World of 5.54% with only 3.56% extra annualised volatility.

Despite the average return being reduced compared to the MSCI EM equity index (13.11% compared to 13.37%), this dynamic allocation strategy uses its risk budget efficiently, providing a more than 3 fold increase in *relative* Calmar ratio[7], being 0.259 on the MSCI EM index, and 0.915 on our strategy.

It is interesting to note that the MSCI EM relative maximum drawdown of -51.68% occurs during a long and steady bull market of developed markets, from Sep 1994 to Aug 1998, as is observable on Figure 1. It is worth noting as well that the underperformance of the emerging market index crosses the 15% limit during 6 different periods in the sample, thus it should not be considered a rare event.

Despite emerging markets offering great perspectives in the long run, they have suffered significant drawdown phases while developed counterparts were experiencing strong growth periods. To illustrate this point, consider an investor that, noticing the strong and persistent outperformance of emerging markets index relative to developed markets one over the years preceding 1993 (left panel of Figure 2), decides to get a strong exposure to this lucrative market. The right panel of Figure 2 plots the cumulative return of the dynamic allocation strategy compared to the emerging and developed markets indices, over the next 6 years up to December 1999 (this zoom period starts about two years before and ends about one year after the maximum RMDD period).

[7] Relative Calmar ratio is a modified version of the traditional Calmar ratio, which replaces the maximum drawdown with the relative maximum drawdown in its calculation.

Over this period, the investor is confronted with a situation in which investing in the emerging markets index without maintaining a dynamic exposure to the developed benchmark leads to a hard-to-recover under-performance. We see clearly from the right panel of Figure Figure 1 that maintaining a dynamic control of relative drawdown to a benchmark would have limited the benchmark underpeformance, avoiding the drawdown while accessing most of the upside of the benchmark. While annualised return of the MSCI World and the dynamic allocation strategy over the period would have been 18.31% and 19.10% respectively, the MSCI EM only returned 8.12%.

Conclusion

In this paper we introduce three modified versions of the maximum drawdown measure and its corresponding dynamic-control strategies, tailored to the needs of different investment objectives.

By limiting the look-back period of the drawdown calculation, a strategy with our modified floor suits risk-averse investors with a limited risk budget, providing a reduction in opportunity costs with respect to the traditional TIPP approach. This method is particularly useful for open funds that can use this technique to guarantee a maximum value loss limit over their recommended investment horizon.

In another attempt to avoid the important opportunity costs embedded in a strict control of the MDD of a portfolio, we introduce the concept of "excess maximum drawdown" and a dynamic allocation strategy that limits this new risk metric for the portfolio. Limiting the *excess* MDD, which is defined as the difference between the MDD of the strategy and the MDD of any reserve asset, avoids holding cash in the portfolio. This alternative risk-management objective is of particular interest in a context of low interest rates and for investors seeking to hedge long term commitments, for whom holding cash represents a "risky" choice.

Furthermore, we introduce the concept of relative maximum drawdown, suitable for investment managers and investors seeking to limit maximum benchmark underperformance. The relative maximum drawdown floor we use, insures a portfolio against benchmark underperformance in any market condition - in bear markets as well as in bull markets - where active managers tend to experience the most severe drawdowns relative to their benchmarks.

End Notes

Details concerning data in tables

- Figures are gross of management fees
- All return and volatility statistics are annualised
- Average returns are computed using geometric average
- Excess return (volatility) to benchmark is computed as the annualised portfolio return (volatility) minus the annualised benchmark return (volatility)
- The maximum drawdown figures are computed on data over a one-year trailing period

Data sources

1. **Thomson Reuters DataStream**

   MSCI WORLD U$ - TOT RETURN IND
   Code: MSWRLD$(MSRI)
   Freq: Monthly

   MSCI EM - TOT RETURN IND
   Code: MSEMKFL(MSRI)
   Freq: Monthly

   JPM EU EMBI GLB DIVS COMPOSITE - TOT RETURN IND
   Code: JPMETOC(RI)
   Freq: Monthly

2. **Kenneth R. French Data Library**

   http://mba.tuck.dartmouth.edu/pages/faculty/ken.french/data_library.html

   US Large Cap (all NYSE, AMEX and NASDAQ stocks)
   Freq: Monthly

   One-month U.S. Treasury Bills
   Freq: Monthly